Linear Decision Rules with Performance Guarantees for Scalable Multi-Stage Generation Planning

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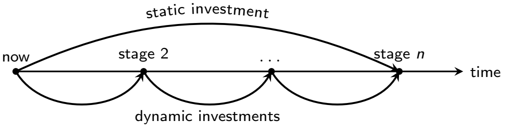
Challenges in long-term power system planning

- Planning power systems for decades ahead involves **uncertainties**

- Static versus Dynamic power system planning
  - Static planning targets a specific year in future (answers "**What**" question)
  - Dynamic planning allocates investments over time (answers "**How**" question)



  - Dynamic planning takes advantage of recourse as uncertainty gradually realizes

- Dynamic planning wins efficiency[1] but requires advanced decomposition, e.g., SDDP, Progressive Hedging, etc.

[1] Beste Basciftci, Shabbir Ahmed, and Nagi Gebraeel. "Adaptive two-stage stochastic programming with an application to capacity expansion planning". In: *arXiv preprint arXiv:1906.03513* (2019).

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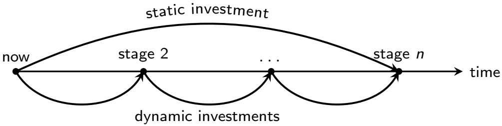

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Can we solve multi-stage investment problems without resorting to decomposition?

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Generation planning problem

$$\min_{\bar{y},y,p} \quad \sum_{t=1}^{T} \left(q_t^\top \bar{y}_t + \sum_{h=1}^{H} c_{\mathrm{e}}(p_{th}) + c_{\mathrm{c}}(y_{th}) \right)$$

Total investment and operating costs

s.to.

$$\mathbb{1}^\top \left(p_{th} + y_{th} - k_h^\ell \circ \ell_t \right) = 0$$

$$\left| F \left(p_{th} + y_{th} - k_h^\ell \circ \ell_t \right) \right| \leqslant \bar{f}$$

Optimal power flow constraints

$$\mathbb{0} \leqslant p_{th} \leqslant k_h^{\mathrm{e}} \circ \bar{p}$$

$$\mathbb{0} \leqslant y_{th} \leqslant k_h^{\mathrm{c}} \circ \sum_{\tau=1}^{t} \bar{y}_\tau$$

Generation limits

$$p_{th} - p_{t(h-1)} \leqslant r_{\mathrm{e}}^{+} \circ \bar{p}$$

$$p_{th} - p_{t(h-1)} \geqslant -r_{\mathrm{e}}^{-} \circ \bar{p}$$

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Ramping limits

$$\forall t = 1, \ldots, T, \quad h = 1, \ldots, H$$

T inv. stages, H operating conditions

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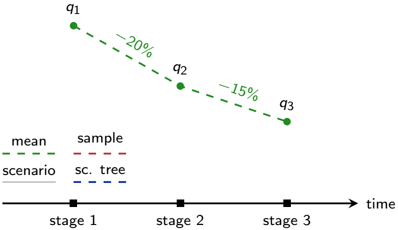
***T* inv. stages, *H* operating conditions**

Addressing Uncertainty using Linear Decision Rules

Investment cost dynamic



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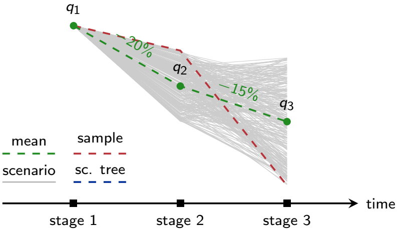

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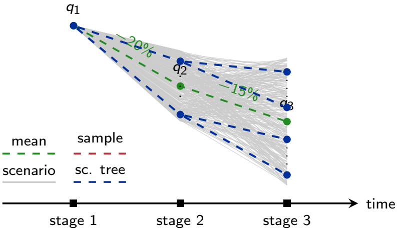
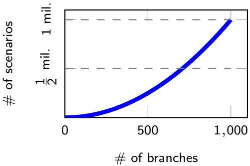


Scenario-based optimization requires:

- Many scenarios to represent uncertainty
- Significant computational effort



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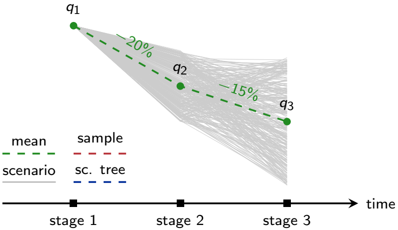

- Let $\boldsymbol{\xi}^t = (1, \boldsymbol{\xi}_2, \ldots, \boldsymbol{\xi}_t)$ be random variables associated with stages 1 to t
- Stochastic cost dynamic from above:

$$q_3(\boldsymbol{\xi}^3) = q_1 - 0.2\boldsymbol{\xi}_2 - 0.15\boldsymbol{\xi}_3$$

2

- Linear decision rule at stage t:

$$\overline{y}_t(\boldsymbol{\xi}^t) = \overline{Y}_t \boldsymbol{\xi}^t,$$

where matrix $\overline{Y}_t$ is an opt. variable

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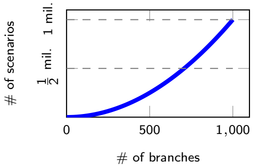
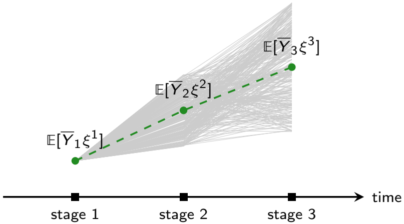


Linear Investment Decision Rules[1]



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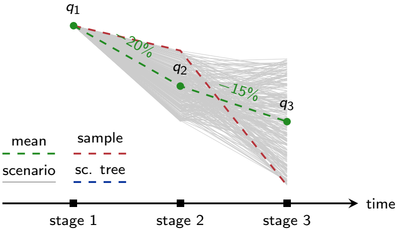

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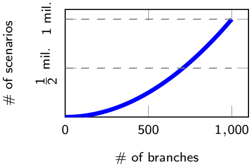

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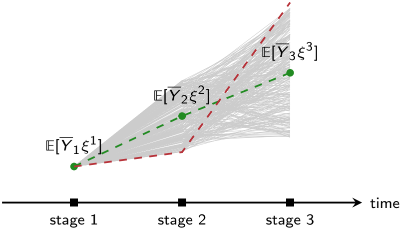

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Performance Guarantees of LDRs: Feasibility

Chance-constrained optimization of investment ($\overline{Y}_t \xi^t$) and operating ($P_t \xi^t$) linear decision rules:

$$
\begin{aligned}
\min_{\overline{Y}_t, \overline{P}_t} \quad & \mathbb{E}_{\mathbb{P}_\xi}\left[\sum_{t=1}^{T} (Q_t \xi^t)^\top \overline{Y}_t \xi^t + (C_t \xi^t)^\top P_t \xi^t\right] && \textbf{Investment + operating costs} \\
\text{s.to.} \quad & \mathbb{P}_{\xi^t}\left[P_t \xi^t - L_t \xi^t \in \text{OPF eq.}\right] \geqslant 1 - \varepsilon_{\text{opf}}, \ \forall t && \textbf{OPF equations} \\
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& \mathbb{P}_{\xi^t}\left[0 \leqslant \overline{Y}_t \xi^t \leqslant \overline{y}^{\max}\right] \geqslant 1 - \varepsilon_{\text{inv}}, \ \forall t && \textbf{Investment limits}
\end{aligned}
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- Includes uncertainty of investment ($Q_t \xi^t$) and operating costs ($C_t \xi^t$), and system load ($L_t \xi^t$)
- **Guarantees** feasibility up to prescribed parameters ε_{opf}, ε_{gen} and ε_{inv}
- Requires uncertainty movements (mean & covariance)

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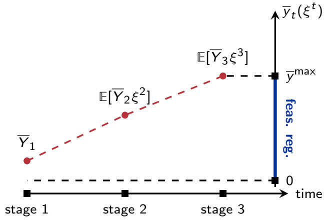
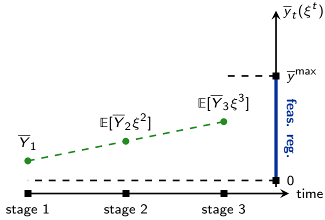
Bad investment decision rules
($1-\varepsilon_{\text{inv}} = 0.5$)



Good investment decision rules
($1-\varepsilon_{\text{inv}} = 0.95$)



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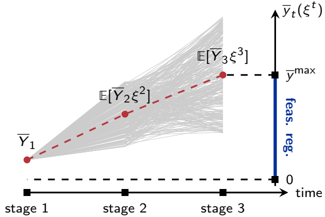

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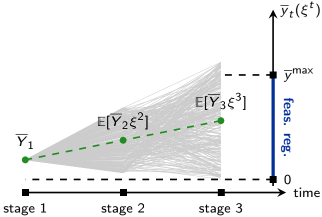

Performance Guarantees of LDRs: Minimal Variability

Variance-aware optimization of linear decision rules:

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$$+\Psi\sum_{t=2}^{T}\text{Var}\left[\overline{Y}_t\xi^t\right] \qquad \textbf{Variability penalty}$$

$$\text{s.to.} \quad \mathbb{P}_{\xi^t}\left[P_t\xi^t - L_t\xi^t \in \text{OPF eq.}\right] \geqslant 1 - \varepsilon_{\text{opf}},\ \forall t \qquad \textbf{OPF equations}$$

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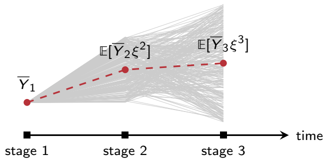
- Additionally **penalizes** the variability of investment decisions (at the expense of exp. costs)

Variability-agnostic investment rules



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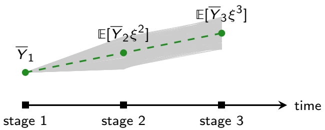
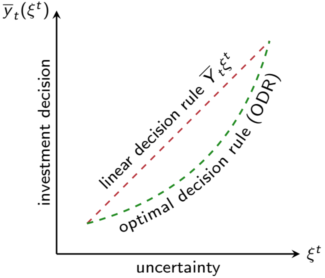


Performance Guarantees of LDRs: Sub-optimality bound

- Restricting decisions to linear functions incurs optimality loss w.r.t. **ideal** stochastic solution
- **$59 billion:** investment into renewable energy in US in 2019
- Even **1%** sub-optimality gap results in the annual loss of **$590 mil.**



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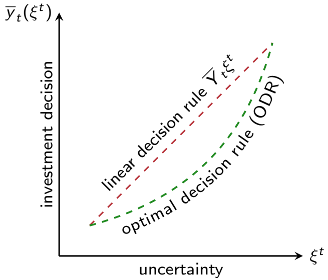
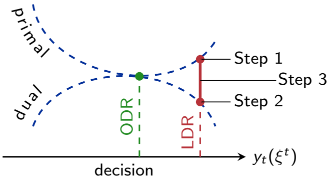


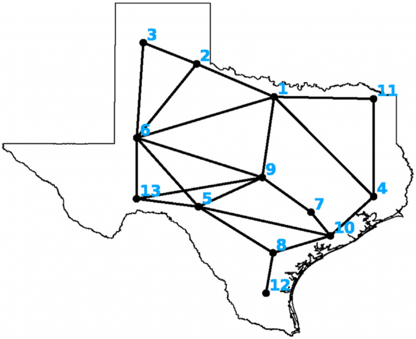
Duality-inspired global sub-optimality gap[3]

- Step 1: Solve primal stochastic program in LDRs (primal gap)
- Step 2: Solve dual stochastic program in LDRs (dual gap)
- Step 3: The difference between the two is the **global** sub-optimality bound



[3] Daniel Kuhn, Wolfram Wiesemann, and Angelos Georghiou. "Primal and dual linear decision rules in stochastic and robust optimization". In: *Mathematical Programming* 130.1 (2011), pp. 177–209.

Numerical Demonstration: Electric Reliability Council of Texas



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- NREL ATB technology baseline
- 5-stage planning horizon
- 24 operating conditions

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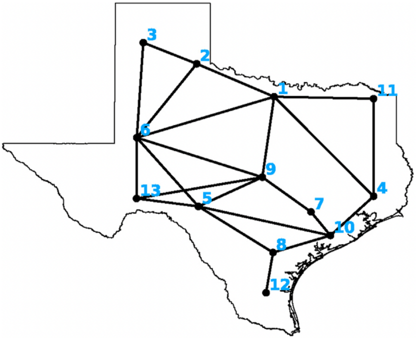

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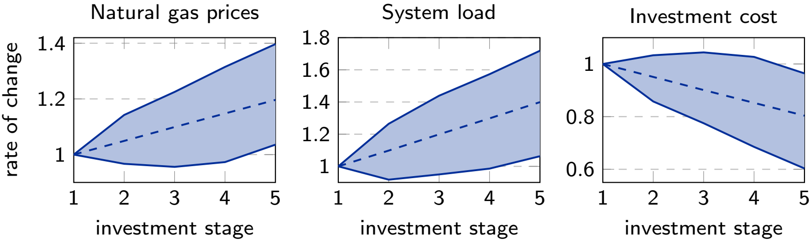

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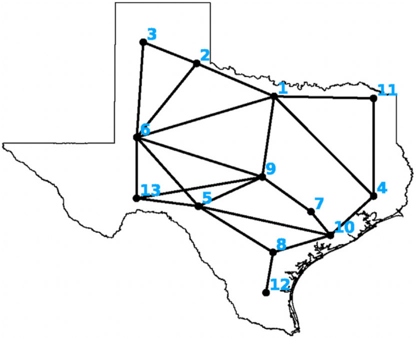

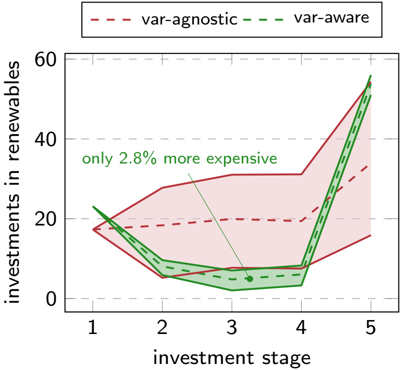

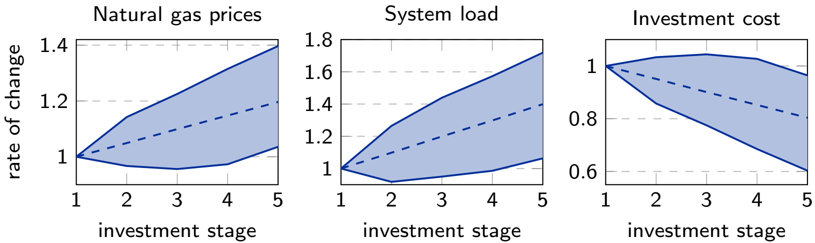

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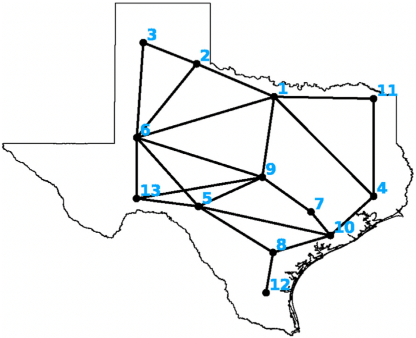

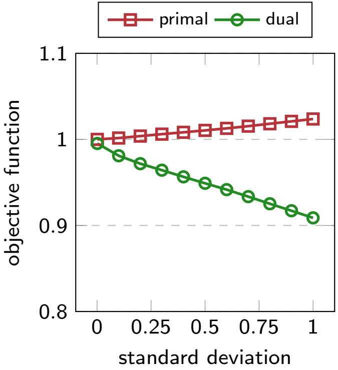

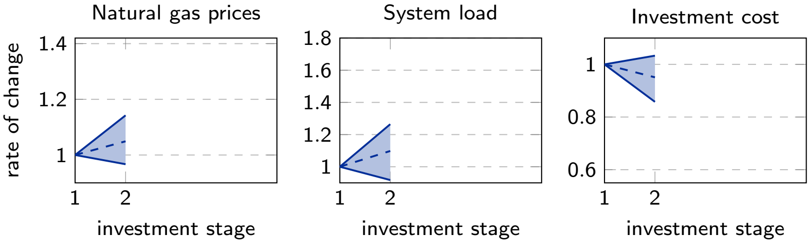

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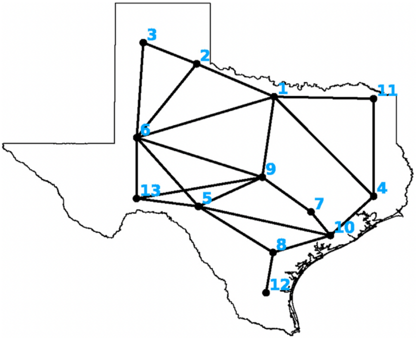

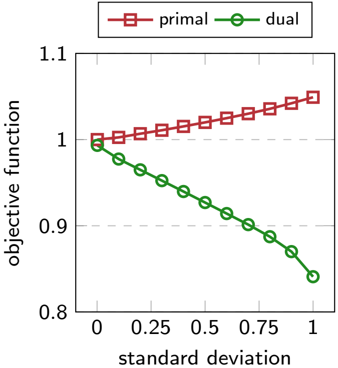

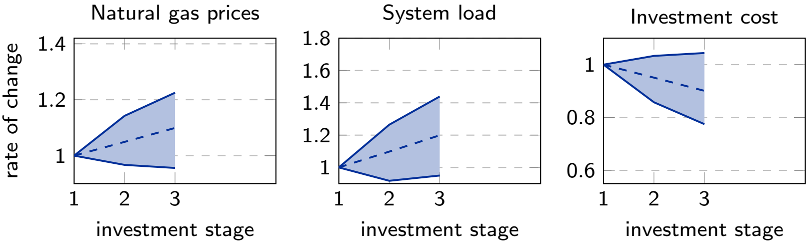

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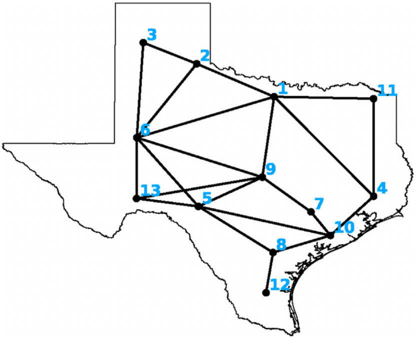
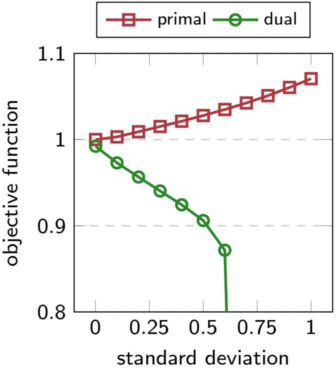
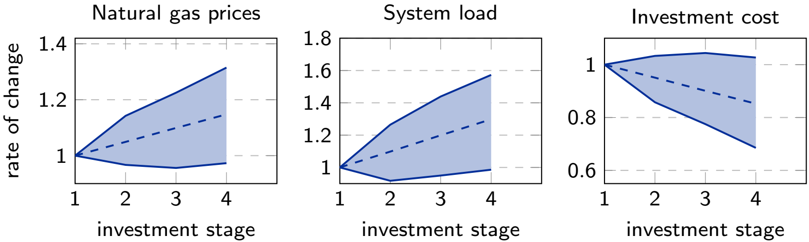
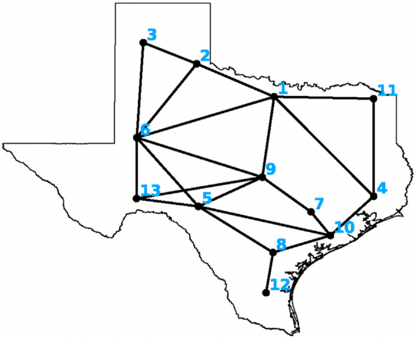
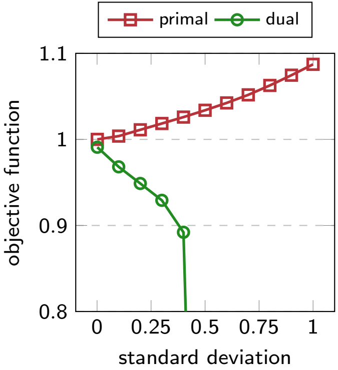


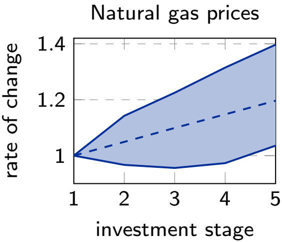


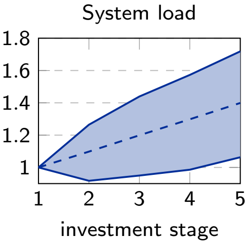


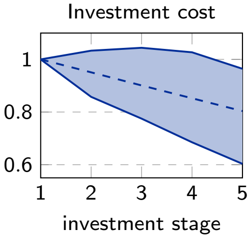
Numerical Demonstration: Electric Reliability Council of Texas











Conclusions

- Long-term power system planning is challenged by uncertainty
- Linear decision rules approximate optimal multi-stage planning under uncertainty
  - Scalability (scenario-free)
  - Feasibility guarantees
  - Decision variability control
  - Sub-optimality bounds
- How good is this approximation? ERCOT system:
  - Good when the system operator has a clear understanding of future trends (sub-optimality $\approx 1\%$)
  - Bad when the system has no idea where the system is going (sub-optimality $\geqslant 12\%$)

Linear Decision Rules with Performance Guarantees for Scalable Multi-Stage Generation Planning

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